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NON-STEADY ONE-DIMENSIONAL GROUNDWATER FLOW IN CONFINED AQUIFER INDUCED BY TIME DELAYED WATER LEVEL CHANGES IN BOUNDING CHANNELS

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ABSTRACT
Unsteady state one dimensional flow of ground water induced by changes in water level in two channels bounding an isotropic homogeneous confined aquifer was studied. The water level in the two channels varies gradually with time and defined by mathematical functions with inbuilt time delay parameters. Exact solution to the problem was sought analytically, using the Laplace transform method. Results were obtained for pressure heads in error functions at various times and places in the aquifer. The equations obtained in the study were in agreement with those obtained by other researchers in previous studies. Using a numerical example, actual head values were determined for varying set values of delay parameters and these were plotted and compared.

Key words
Confined aquifer, channel water level, isotropic, transmissibility, Laplace transform, error functions, delay parameters.

INTRODUCTION
Many situations do occur in the field in which a body of water in a channel interfacing an aquifer would constitute major sources of ground water recharge and flow in the aquifer. The water level in the channel could change either rapidly or slowly, depending on the pattern of pressure force causing the change. For instance, sudden release of water from remote sources due to water leakages or flood waves caused by intense precipitation of short duration would generate large surface flow leading to rapid build-up of water level in the river channel. Similar situations would be expected to occur, from either pumping operations in irrigation fields or, rapid withdrawal of water from surface reservoirs. In some instances, there would be gradual build-up of water level in reservoirs such as during operations in pumped-storage hydropower generation plants.

On the other hand, the drying up of lakes and reservoirs adjoining an aquifer in dry regions due to high evaporation processes would also lead to slow decline in water level. On the global scale, there would be expected, gradual change in water level, resulting from freezing and de-freezing of polar ice, caused by climate change. In coastal areas adjoining permeable rocks, flood waves generated by tidal oceans would raise the water level and induce ground water flow in aquifers. Although the changes in water level discussed herein take place in remote parts of the aquifer; however, the effects are transmitted either slowly or rapidly to all parts of the aquifer system, depending on the nature of change.

The general problems of water level changes in channels, lakes and lagoons bounding confined aquifers have been studied by many hydrologists and researchers. There was also study of the groundwater flow in African Saharan aquifer system (Hammad, 1969) among many other studies in Africa. The results of the study established the pattern and general trend of ground water flow in the sub-Saharan region. Later (Marino,1974; Gill,1984), analytical solutions were derived for groundwater flow problems in confined and semi-finite aquifers due to changes in channel water levels. Thereafter (Mustafa, 1987, 2013), similar approaches were employed and in addition, there was introduced a practical dimension to the problems, by examining variations in groundwater flow induced by both surface infiltration/evaporation and changes in water level in bounding channels.

With the recent advances in the application of GIS system in watershed modelling, 3-D groundwater flow modelling gained prominence, as more information can be obtained and processed on regional scale, on the nature, pattern, distribution and trend of groundwater flow in aquifers. There was applied (Gossel et al (2004) for instance, the GIS-based groundwater flow modelling technique to study the long-term groundwater flow in the Nubian sandstone aquifer in Eastern Sahara. The results of the studies established that, the Nubian Aquifer groundwater system, had been formed largely, by infiltration, during the 20,000-5,000(BP). The studies further showed that, the aquifer system, is a fossil aquifer and had been in unsteady state condition for the past 3,000 years.

Of recent, the problem of surface and subsurface water interaction generally observed in coastal areas giving rise to moving water boundary was studied (Kong et al.,2010). In the study, a moving water boundary was simulated by a ground water and surface water 2-D model. The resulting governing equations were however not amenable to analytical solution but, were solved numerically, using finite difference methods.

On regional scale, due to its complexity, regional groundwater flow is usually studied by hydrologic mathematical catchment models. The most widely used groundwater model is, USGS finite different based MODFLOW. The versatility of the model has been enhanced greatly upon its successful integration with the Geographic Information System (GIS).

The MODFLOW-2000 successfully employed (Hashemi et al., 2012) to study ground water flow under steady state conditions, to determine the recharging system in the Gareh-Bygone Plain in southern Iran. The study used the flood water spreading system that was established to recharge the ground water. The results established that, without surface water inflow, the plain was being recharged through a fault conducting water from the upper sub-basin.

Also (Khadri and Pande, 2016) recently, the Mahesh River Basin in the Akola and Buddha districts of India was modelled, using MODFLOW model, in which, it was established that, the aquifer system was stable under the conditions prevailing at the time.

Using the similar approach (Baalousha, 2016), MODFLOW model was successfully applied, to study groundwater flow for Qatar aquifers comprising of karst limestone containing cavities, sink holes and depressions, covering the country's area of 11,586km^2. The study estimated the amount of recharge and established the trend of groundwater flow, which was observed to be decreasing over the years. The study also established that, sea water intrusion was occurring in the coastal areas and that, there was lateral flow into Qatar, through its southern border with Saudi Arabia.

In a similar study (Aniekan et al.,2014), MODFLOW model was applied to determine the mode and pattern of groundwater flow in the coastal aquifers of Akwa Ibom state, Nigeria. The study evaluated the recharge values for the six different zones studied and established that, there was high recharge occurring in the area and thus, showing high potential for ground water resources.

Most groundwater flow modelling studies are largely carried out under the steady state flow conditions. However, it is observed that, on the contrary, groundwater flow in the field is generally unsteady and transient.

Few researchers of recent (Hong Niu et.al., 2015) employed analytical methods to study the flow of ground water under unsteady steady state. The findings on the whole, showed that, when time was large enough, the flow distribution under unsteady state conditions tended to the steady state flow.

The problem presented in this study is, aimed at determining the pattern of flow resulting from time delayed water level changes taking place in two channels bounding a semi-finite artesian aquifer. A mathematical representation of the problem is shown in Fig.1 for which analytical solution was sought.

MATHEMATICAL REPRESENTATION OF THE PROBLEM

In the study presented, a situation is visualized in which, the water levels in both the LHS and RHS channels shown in Fig.1 were initially at the same level, the datum. Thereafter, both levels change gradually, as to induce ground water flow into the adjoining aquifer.

A solution is sought on the nature and distribution pattern of groundwater flow resulting from the water level changes under unsteady state conditions using boundary conditions specified by mathematical functions with inbuilt delay parameters.

The generalized ground water flow equation results from combining continuity equation (mass balance) with Darcy's law. Thus, for inhomogeneous and anisotropic confined aquifer, the continuity equation(Trescott and Larson, 1977) is,

$$-\left\{\frac{\partial}{\partial x}(q_x) + \frac{\partial}{\partial y}(q_y) + \frac{\partial}{\partial z}(q_z)\right\} + Q = S_s\frac{\partial h}{\partial t} \quad (1)$$

Q is pumping or injection of a vol. of flux (L^3/T per vol.(L^3)) and S_s is specific storage (L^{-1}).
Combining Equation (1) with Darcy's law $\{q_x = -K_x \frac{\partial h}{\partial x}$, $q_y = -K_y \frac{\partial h}{\partial y}$ and $q_z = -K_z \frac{\partial h}{\partial z}\}$ is obtained, for homogeneous hydraulic conductivity, i.e. independent of x, y and z, anisotropic confined aquifer without pumping or recharge

$$K_x\,\partial^2h/\partial x^2 + K_y\,\partial^2h/\partial y^2 + K_z\,\partial^2h/\partial z^2 = S_s\,\partial h/\partial t \quad (2)$$

For homogeneous and isotropic aquifer, i.e., $K_x = K_y = K_z = K$, Eq. (2) reduces to

$$\partial^2h/\partial x^2 + \partial^2h/\partial y^2 + \partial^2h/\partial z^2 = (S_s/K)\,\partial h/\partial t \quad (3)$$

Thus, the general groundwater flow equation for homogeneous and isotropic aquifer is given by

$$K\nabla^2h = S_s\,\partial h/\partial t \quad (4a)$$

For aquifer of constant thickness b Eq.4a is expressed as

$$b(K\nabla^2h) = b(S_s\,\partial h/\partial t) \quad \text{or}$$

$$T\nabla^2h = S\,\partial h/\partial t \;\rightarrow\; \nabla^2h = (S/T)\,\partial h/\partial t \quad (4b)$$

Such that, T = bK and S = b S_s whereas, S is dimensionless and T(L^2/T)

∇^2 is the Laplacian operator = $\partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$

For 3- dimensional homogeneous isotropic and uniform thickness confined aquifer, the groundwater flow equation is given by

$$\partial^2h/\partial x^2 + \partial^2h/\partial y^2 + \partial^2h/\partial z^2 = (S/T)\,\partial h/\partial t \quad (4c)$$

If there is no flow in the vertical direction and flow is only in one dimension, the x axis, Eq.4c reduces to

$$\partial^2h/\partial x^2 = (S/T)\,\partial h/\partial t = (1/\propto)\,\partial h/\partial t \quad (4d)$$

S is the storage coefficient, T is the transmissibility,

$\propto = T/S$, a coefficient , b is aquifer thickness and x, y, z orthogonal direction

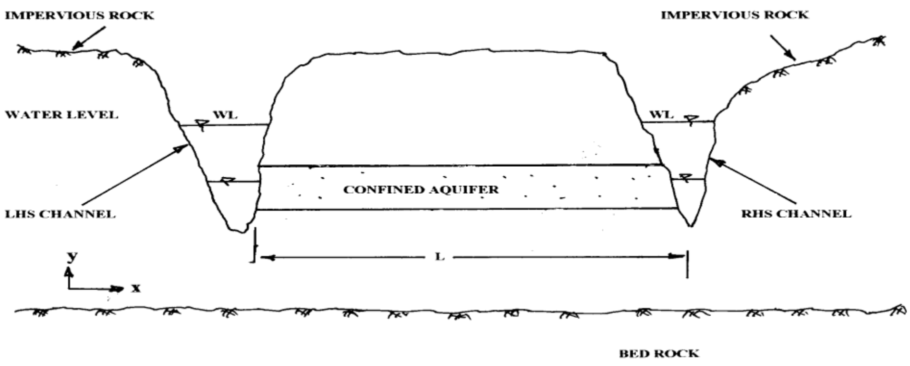

FIG.1 SCHEMATIC REPRESENTATION OF GROUNDWATER FLOW IN THE AQUIFER

Equation (4d) is a Partial Differential Equation (PDE)

In seeking solution to the PDE, the boundary conditions must be specified, in addition to the initial conditions. The following boundary and initial conditions apply herein

$h(x,0) = 0 \qquad t \le 0 \qquad - \qquad (5a)$

$h(0,t) = H_o[1 + \exp(\zeta t)\,\mathrm{erfc}(\sqrt{\zeta t})] \qquad t > 0 \qquad - \qquad (5b)$

$h(L,t) = H_o[1 + \exp(\mu t)\,\mathrm{erfc}(\sqrt{\mu t})] \qquad t > 0 \qquad - \qquad (5c)$

This means, the head h, in the channels at $x = 0$ and at $x = L$ will change after time $t > 0$ while,

ζ and μ are introduced as time delay parameters.

Let us define $\theta^2 = \zeta$ and $\beta^2 = \mu$

Let solution be sought by taking the Laplace transform of Eq.4d, and Eqs.5(a-c) thus:

$L\ \{h(x,t)\} = h(x,p)$

where p is the transform time

Hence, the solution for Eq.4d is put in the form:

$h(x, p) = C_1\cosh(\varphi x) + C_2\sinh(\varphi x) \qquad - \qquad (6)$

where $\varphi = (p/\alpha)^{1/2}$ and C_1 and C_2 are constants

Now, from Eq. (5b): $h(0, p) = H_0\left\{\frac{1}{p} + \frac{1}{\sqrt{p}(\sqrt{p}+\theta)}\right\} \qquad - \qquad (7a)$

and from Eq. (5c): $h(L, p) = H_0\left\{\frac{1}{p} + \frac{1}{\sqrt{p}(\sqrt{p}+\beta)}\right\} \qquad - \qquad (7b)$

Putting these into Eq.6, the values of the constants are obtained as

$C_1 = H_0\left\{\frac{1}{p} + \frac{1}{\sqrt{p}(\sqrt{p}+\theta)}\right\}$ and $C_2 = H_0\left[\left\{\frac{1}{p} + \frac{1}{\sqrt{p}(\sqrt{p}+\theta)}\right\} - \left\{\frac{1}{p} + \frac{1}{\sqrt{p}(\sqrt{p}+\beta)}\right\}\frac{\cosh(\varphi L)}{\sinh(\varphi L)}\right] \quad - \quad (8)$

For brevity, let $\phi_1 = \left\{\frac{1}{p} + \frac{1}{\sqrt{p}(\sqrt{p}+\theta)}\right\}$ and

$\phi_2 = \left\{\frac{1}{p} + \frac{1}{\sqrt{p}(\sqrt{p}+\beta)}\right\}$ such that,

$C_1 = \phi_1$ and $C_2 = \left(\frac{1}{\sinh(\varphi L)}\right)\{\phi 2 - \phi 1\cosh(\varphi L)\}$

After rearranging Eq.6 after putting the values of the constants C_1 and C_2,

$h(x, p) = H_0\left\{\phi 1\,\frac{\sinh\varphi(L-x)}{\sinh(\varphi L)} + \phi 2\,\frac{\sinh(\varphi x)}{\sinh(\varphi L)}\right\} \qquad - \qquad (9)$

Now, using Binomial series expansion and after some algebraic rearrangement of Eq.6 gives for the term,

$\phi_1 \frac{\sinh \varphi(L-x)}{\sinh(\varphi L)} = \phi 1 \{\sum_{n=0}^{\infty}[\exp -\varphi(2nL-x)] + \sum_{n=0}^{\infty}[exp - \varphi(2nL+x)]\}$

\- (10a)

And the term,

$\phi 2 \frac{\sinh(\varphi x)}{sinh(\varphi L)} = \phi 2\{\sum_{n=0}^{\infty}[\exp -\varphi(2nL+L-x)] + \sum_{n=0}^{\infty}[exp - \varphi(2nL+L+x)]\}$ - (10b)

h(x,t) = L $^{-1}$ {h(x,p) }

For the term with ϕ_1 , the Inverse Laplace transform is

$L^{-1}\{\phi_1 \frac{\sinh \varphi(L-x)}{\sinh(\varphi L)}\}$ which is rewritten as

$L^{-1}\left\{\frac{1}{p} + \frac{1}{\sqrt{p}(\sqrt{p}+\Theta)}\right\}\{\sum_{n=0}^{\infty} [\exp -\varphi(2nL-x)] + \sum_{n=0}^{\infty} [exp - \varphi(2nL+x)]\}$

For the terms with $\frac{1}{p}$ upon inverting and replacing φ by $\sqrt{\alpha t}$, the solution is given by

$\sum_{n=0}^{\infty}\{ \text{erfc}((2nL- x)/2\sqrt{\alpha t})\} + \sum_{n=0}^{\infty} \{ \text{erfc}((2nL+x)/2\sqrt{\alpha t}))\}$ - (11a)

Similarly, for the terms with $\frac{1}{\sqrt{p}(\sqrt{p}+\Theta)}$ upon inversion and replacing

the values of Θ, $\boldsymbol{\beta}$ with those of ζ and μ respectively, give

$\sum_{n=0}^{\infty}\exp [\zeta t + (2nL-x)\sqrt{\zeta /\alpha}] \sum_{n=0}^{\infty} \text{erfc} [(2nL-x)/2\sqrt{\alpha t} + \sqrt{\zeta t}]$

$+ \sum_{n=0}^{\infty}\exp [\zeta t + ((2nL + x)\sqrt{\zeta /\alpha})] \sum_{n=0}^{\infty} \text{erfc}[(2nL+x)/2\sqrt{\alpha t} + \sqrt{\zeta t}]$ - (11b)

Similar approach for the terms with ϕ_2 , taking the inverse transform:

$L^{-1}\left\{\phi 2 \frac{\sinh(\varphi x)}{sinh(\varphi L)}\right\} = L^{-1}\left\{\frac{1}{p} + \frac{1}{\sqrt{p}(\sqrt{p}+\beta)}\right\}\left\{\begin{matrix} \sum_{n=0}^{\infty}[\exp -\varphi(2nL+L-x)] \\ + \sum_{n=0}^{\infty}[exp - \varphi(2nL+L+x)]\end{matrix}\right\}$

For the terms with $\frac{1}{p}$ would give

$\sum_{n=0}^{\infty}\text{erfc}[((2n+1)L-x)/2\sqrt{\alpha t}] + \sum_{n=0}^{\infty}\text{erfc}[((2n+1)L+x)/2\sqrt{\alpha t}]$ - (11c)

And for the terms with $\frac{1}{\sqrt{p}(\sqrt{p}+\beta)}$ would give

$\sum_{n=0}^{\infty}\left[\exp[\mu t + (2n+1)L - x]\sqrt{\mu/\alpha}\right] \sum_{n=0}^{\infty} \text{erfc}[((2n+1)L-x)/2\sqrt{\alpha t} + \sqrt{\mu t}]$

$+ \sum_{n=0}^{\infty}\left[\exp[\mu t + (2n+1)L + x)]\sqrt{\mu/\alpha}\right] \sum_{n=0}^{\infty} \text{erfc} [(2n+1)L+x)/2\sqrt{\alpha t} + \sqrt{\mu t})]$

\- (11d)

The total solution for head h(x, t) is obtained by adding Eq.11(a-d) to give

$h(x,t)= H_0 \{\sum_{n=0}^{\infty} \text{erfc} [(2nL- x)/2\sqrt{\alpha t}] + \sum_{n=0}^{\infty} \text{erfc} [(2nL+x)/2\sqrt{\alpha t}]$

$+ \sum_{n=0}^{\infty}\exp [\zeta t + (2nL-x)\sqrt{\zeta /\alpha}] \sum_{n=0}^{\infty} \text{erfc} [(2nL-x)/2\sqrt{\alpha t} + \sqrt{\zeta t}]$

$+ \sum_{n=0}^{\infty}\exp [\zeta t + (2nL + x)\sqrt{\zeta /\alpha}] \text{erfc}\sum_{n=0}^{\infty} [(2nL+x)/2\sqrt{\alpha t} + \sqrt{\zeta t}]$

$+ \sum_{n=0}^{\infty}\text{erfc}[((2n+1)L-x)/2\sqrt{\alpha t}] + \sum_{n=0}^{\infty}\text{erfc}[((2n+1)L+x)/2\sqrt{\alpha t}]$

$+ \sum_{n=0}^{\infty}\left[\exp[\mu t + (2n+1)L - x]\sqrt{\mu/\alpha}\right] \sum_{n=0}^{\infty} \text{erfc} [((2n+1)L-x)/2\sqrt{\alpha t} + \sqrt{\mu t}]$

$+ \sum_{n=0}^{\infty}\left[\exp[\mu t + (2n+1)L + x)]\sqrt{\mu/\alpha}\right] \sum_{n=0}^{\infty} \text{erfc} [((2n+1)L+x)/2\sqrt{\alpha t} + \sqrt{\mu t}] \}$

\- (12)

where,

erf(x) = 1 – erfc(x) and $\text{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e - \tau \, dt$ - (13a)

where $\tau = t^2$

The values of the function can be obtained in standard mathematical handbooks.

Using Taylor series expansion and integrating the error function, gives

$\text{erf}(x) = 1-\frac{1}{\sqrt{\pi}}-e^{x2} \{1/x - 2/x^3 + (1.3)/(2^3x^5) - (1.3.5)/(2^3x^7) + \ldots\ldots\}$ - (13b)

From Abramowitz and Stegun (1992), an approximation of the function is

$\text{erf}(x) = 1 - 1/(1 + a_1x + a_2x^2 + a_3x^3 + a_4x^4)^4 + e(x)$ - (13c)

Such that, $|e(x)| \leq 5 \text{ X } 10^{-4}$

Constants a_1, a_2, a_3 and a_4 are obtained from rational fitting of weights thus:

a_1= 0.278393, a_2= 0.23030389, a_3 = 0.000972 and a_4 = 0.078108
The error function complimentary takes special values at
erfc(0) =1 ; erfc (∞) =0 and erfc(−∞) = 2

Discussion of Results

In the case the delay constants ζ and μ take zero values, that is $\zeta = \mu = 0$
Eq.12 simply reduces to

$$h(x,t)=2 H_0 \{\sum_{n=0}^{\infty} \text{erfc}\,[(2nL- x)/2\sqrt{\alpha t}\,] +\sum_{n=0}^{\infty} \text{erfc}\,[(2nL+x)/2\sqrt{\alpha t}\,] + \sum_{n=0}^{\infty} \text{erfc}\,[((2n+1)L-x)/2\sqrt{\alpha t}] + \sum_{n=0}^{\infty} \text{erfc}[((2n+1)L+x)/2\sqrt{\alpha t}]\,\} \quad - \quad (14)$$

Which means the flow is caused by head $H_o$only at both left and right-hand side boundaries, giving results similar to the ones and obtained in earlier studies (Gill ,1984 and Mustafa, 1987- 2013).

Numerical Example

A numerical example is given for the solution in which typical aquifer values and constants were assigned to parameters in Eq.12. To obtain the head distribution in the aquifer at various time intervals, the distance between the channels is set at, **L**= 1000m. Time t is practical time in days and set at, t = 1 day, 2 days, 5 days, 100, days, 200 days, . . 1000 days. The head causing flow H_o is fixed at, 10m. This value is sufficient to generate determinable flow in the aquifer system.
Using a typical aquifer parameter $\boldsymbol{\alpha}$ = 12,000m^2/day, the value for the delay constant at a start was put at ɵ = 0.000001 and then increased to 0.000005, 0.00005 and 0.005. Likewise, the other delay parameter **β** was fixed at a value 0.0001 and then increased to 0.01. Using MS- EXCEL, the various heads were calculated at distances fixed at x = 10m, 200m, 400m, 600m, . . 1000m from Eq. 12, using the approximation formula obtained for the error function given in Eqs.13 (a-c). The resulting head distributions at different places in the aquifer were evaluated and plotted in Figs.2(a-d) and shown in the Appendix.

The values of head at both left and right-hand side channels are not fixed but controlled by the delay parameters ▯ and **β** as reflected in spread of head. However, it would appear, that delay parameter ɵ tends to be more sensitive and defining the pattern of flow. Overall, the pattern of head distribution in the long run become uniform albeit, with varying magnitudes. The head causing flow is from both boundaries, the direction of flow determined by the delay parameters. At the initial time, the head values do not change appreciably until after considerable time interval as dictated by the delay parameters.

Conclusion

The problem of flow situation in finite artesian aquifer in which the level of water in two channels bounding an aquifer change was investigated. The change takes place gradually, over time, defined by delay parameters; for which, solutions were obtained, using Laplace transform method. Analytical solutions for the problem studied were given in error functions. A numerical example was given in which at various time intervals the head distribution in the aquifer at different places was calculated using MS-Excel computer program. These solutions would help to understand the nature of ground water flow resulting from sudden inflow of surface water, such as caused by delayed flood water, or tidal waves generated in coastal areas.

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Notations

H_0, h = head in aquifer - (L)
α =T/S - (L^2T^{-1})
q_x, q_y, q_z = flux - (LT^{-1})
Q = pumping or recharge flow- (L^3/T per vol. L^3)
K_x, K_y, K_z, K = hydraulic conductivity- (LT^{-1})
T = transmissibility - (L^2T^{-1})
S_s = Specific Coefficient (L^{-1})
S= Storativity of the aquifer - (storage coefficient; dimensionless)
$\varphi = (p/\alpha)^{1/2}$ - (L^{-1})
p = parameter in Laplace transform - (T^{-1})
L = distance separating the two bounding channels - (L)
$\square, \beta$ = time delay parameters - ($T^{-1/2}$)
ζ and μ = *time delay constants - (T^{-1})*
ϕ_1, ϕ_2 *- (T)*
t, τ= time - (T)
C_1 and C_2 are constants in Laplace transform - (LT)

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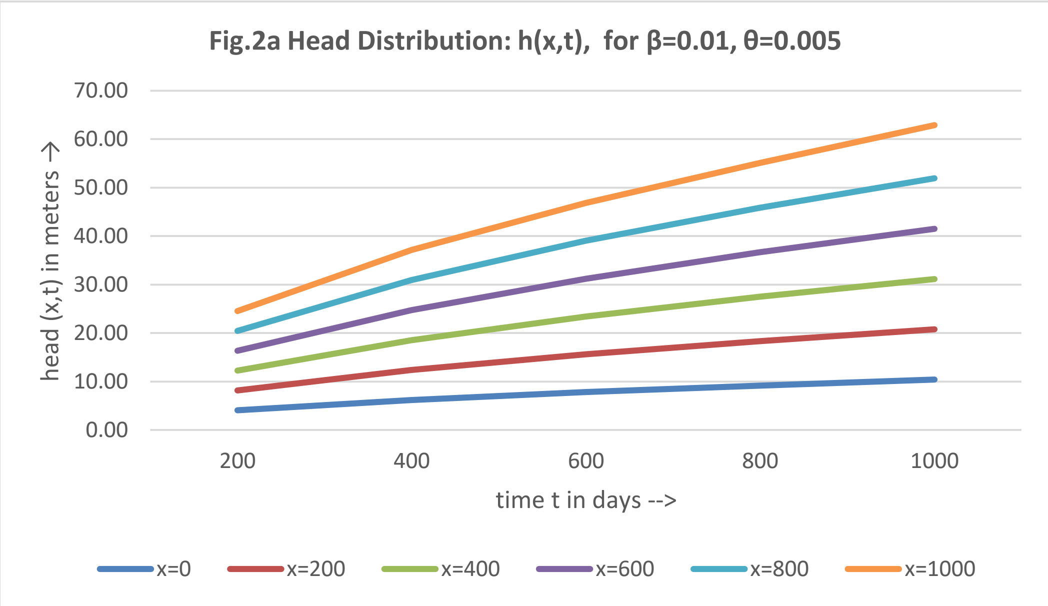
APPENDIX

Figs. 2(a-d) Showing head distribution for various values of delaying parameters ▯ and β



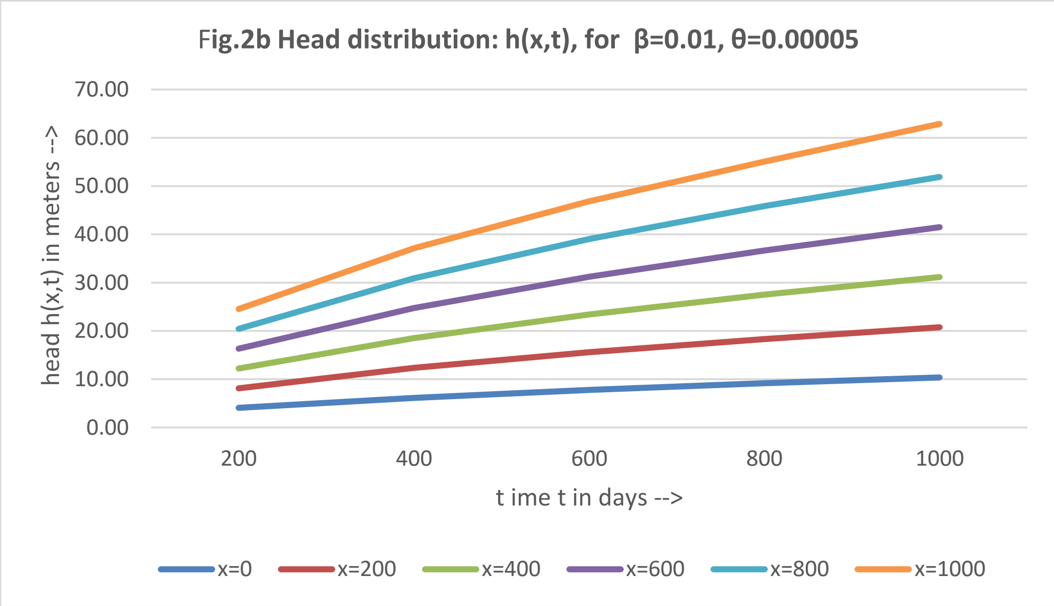


Fig.2b Head distribution: h(x,t), for β=0.01, θ=0.00005
70.00
60.00
50.00
40.00
30.00
20.00
10.00
0.00
head h(x,t) in meters -->
200
400
600
800
1000
t ime t in days -->
x=0
x=200
x=400
x=600
x=800
x=1000

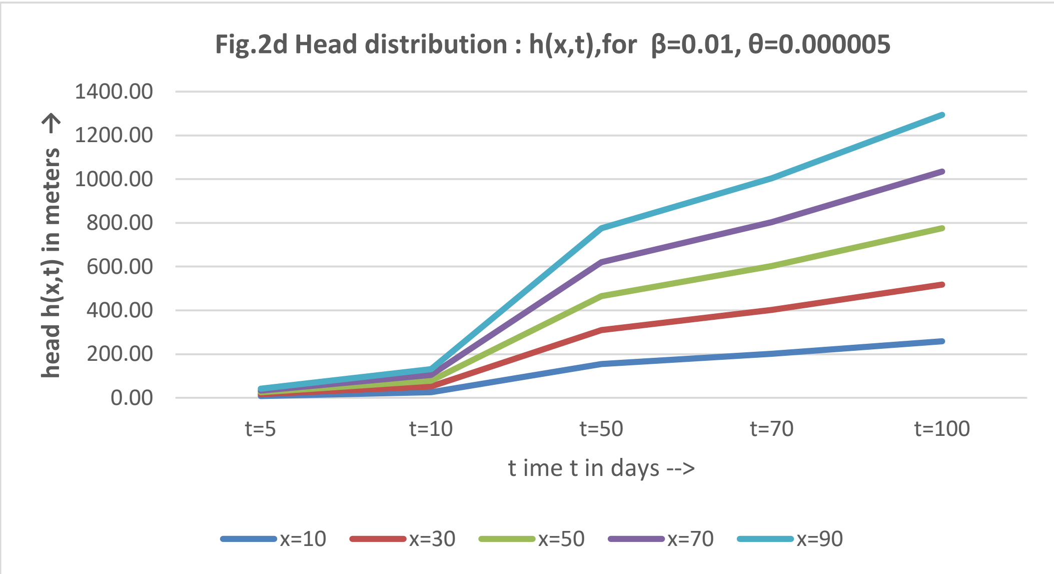


Fig.2d Head distribution : h(x,t),for β=0.01, θ=0.000005
1400.00
1200.00
1000.00
800.00
600.00
400.00
200.00
0.00
head h(x,t) in meters →
t=5
t=10
t=50
t=70
t=100
t ime t in days -->
x=10
x=30
x=50
x=70
x=90