MATHEMATICS, SCIENCE, AND CULTURAL CHANGE

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According to Roger Bacon (1214-1294), "Mathematics is the gate and key of the sciences. Neglect of Mathematics works injury to all knowledge, since he who is ignorant of it cannot know the other sciences or things of this world, and what is worse, men who are thus ignorant are unable to perceive their own ignorance and so do not seek a remedy".

Mathematics enables various branches of science draw implications of their observations and experimental findings. A computer is now an inevitable tool in science; its internal working rests on basic mathematics -arithematic to base$_2$

Mathematics is at the heart of our important scientific theories; Newtonian mechanics, Electromagnetic theory of Maxwell, Einstein's theory of relativity, quantum theory of Planck etc. Further, vast areas of mathematics grew out of efforts to find solutions to problems in science. We give two examples. The first one is about geometry, also sometimes referred to as the gift of the Nile. The story goes that in the 14th Century B.C., King Sesotris divided the land among Egyptians, each receiving a rectangle of same area, and was taxed accordingly. If anyone lost some of his land during the annual overflow of the Nile, he had to report the loss. An overseer would then be sent to measure the loss and make an abatement of tax. Some sources claim that this is the origin of geometry. The second example is more recent; it is about topology, whose origin is associated with a problem on a bridge. During the 18th century, in the German town of Konigsberg (now the Russian city of Kaliningrad), people enjoyed strolling along the banks of the Preger river, which meandered through the town, and was crossed by seven bridges which ran trom each bank of the river to two islands in the river, with a bridge joining the islands. One question asked is as follows:
"How can you take a stroll so that you cross each of the seven bridges exactly once?".

Leonhard Eu!er (1707 -1783) converted this problem to one on vertices and networks in a diagram. He made general discoveries about networks, and found that the answer to the Konigsberg bridge problem, is in the negative, namely, that it is impossible to take a stroll so that each of the seven bridges is crossed only once! In the process of obtaining this result, he originated a new kind Of geometry, a geometry which does not depend on the size or shape of the figure, but about places, and how they are connected by arcs. Out of this grew the branch of mathematics called 'Topology".

According to H.E. Wolfe, (Introduction to Non-Euclidean geometry (1945)),"It has been said that no subject when separated from its history, loses more than Mathematics...........".

In this lecture, we shall discuss the evolution of mathematics from ancient limes to about the last century. This is a vast subject, and it is not possible within the hour to give more than a brief sketch of some of the areas. The choice of material gives an edge to areas associated with the explanation of physical phenomena at the time. Because of such inter-relationships between mathematics and science as already indicated, which inter-relationships date back to early times, we consider that our talk should also touch on some of the related science. We also consider it reasonable that our talk includes reference to the society at a time. It is in this spirit that the title, "Mathematics, Science, and Cultural Change" was chosen.

As far as I know, very few ancient civilizations had what may be regarded as rudiments of mathematics. Examples include Egypt and Babylonia, with civilizations dating back to 4,000 B.C. In their records were developed number systems: whole numbers, fractions, a fair amount of arithmetic, solution of some simple problems involving unknown quantities, and very simple rules for finding areas and volumes of geometrical figures. These simple rules were obtained through accumulated experience and inductive reasoning, and not surprising then, some of them were incorrect. For example, the Babylonian rule for calculating the area of a circle of radius r is $3r^2$, while Egyptian rule is $3.16r^2$. They respectively used, 3, 3.16 in place of π. Thus, one serious weakness in Egyptian and Babylonian mathematics is that the conclusions were obtained empirically. There was hardly any conscious thought about abstraction, nor formulation of general methodology and no concept of proof. There was no conception of a theoretical science.

The Babylonians did more arithmetic and algebra, while the Egyptians did more geometry. The Egyptians were great builders, and applied geometry in the building of their temples and pyramids. Also, by observing the motion of the sun, they ascertained that a year contains 365 days. In this connection, the Babylonians, developed a calendar of a year consisting of 12 months of 30 days each, with a 13th month at the end of every sixth year.

Whatever may be regarded as achievements of Egyptians and Babylonians, there was hardly any discernible search for theoretical systematic knowledge. Partly because of this, it is now usually taken that mathematics as an organized and reasoned discipline did not exist before the lime of the classical Greek civilization, about the sixth century till the third century B.C., continuing during The Alexandrian Greek civilization which lasted till about the seventh century A.D. Greek Mathematics developed in several centres that succeeded one another, each building on the work of its predecessors. At each centre, an informal group of scholars carried on its activities under one or more great leaders. An example is the Pythagorean school noted for its study of the properties of numbers. Others noted for Mathematics and methods of deductive logic include those of Socrates, Plato, Aristotle, all these before Euclid went to Alexandria about the 3rd century B.C. and Archimedes,(287 -212 B.C.).

The Greeks believed that truths came from the mind. These truths were either innate and merely recalled, or they were suggested by experience and immediately recognized as self-evident In Mathematics and Science, the Greeks selected some of these truths or obvious facts as starting points: they are referred to as axioms, in the case of Mathematics. By and large, the Greeks, especially of the classical times, were unwilling to observe and experiment. They regarded this means of obtaining knowledge as unnecessary. The belief that truths can be found by exploring the mind instead of exploring the physical world persisted till modem times.

Having selected their axioms, the Greeks applied the method of deductive reasoning, with these axioms as premises. In this case, given that we accept the premises, then we must accept the conclusion it produces. This method clearly has a serious advantage over reasoning by induction or analogy or obtaining results by trial and error. Also in contrast to observation and experimentation, deduction can be carried out without recourse to equipment - which could be expensive. In some cases, deductive reasoning is the only method available, as for example in the measurement of astronomical distances. The Greeks appreciated the value of persuasive argument to the full, and insisted that all mathematical conclusions must be established only by deductive reasoning. The exclusive use of deduction has served as an important factor which differentiates Mathematics from other fields of knowledge, and has served as a source of strength. According to Hermann Hankel, "In most sciences, one generation tears down what the other has built, and what one established, another undoes. In mathematics alone, each generation builds a new story to the old story". A prominent factor in this assertion is the method of deductive reasoning used in obtaining mathematical results.

In addition to insisting on the deductive method, the Greeks effectively made Mathematics abstract. To the Egyptians for example, a line was no more than either a stretched rope or the edge of a field, and a rectangle was the boundary of a field. In contrast, the Greeks recognized and emphasized that mathematical entities - numbers, geometrical figures - were abstractions, ideas entertained by the mind, and distinguished from physical objects and pictures. In particular, the words point, line, triangle became mental concepts. Since the Greeks reason about general concepts, their conclusion would apply to all objects of which the concepts were representative. Sometimes one would obtain new knowledge relating to the different representatives which experience might not suggest.

(i) The area πr^2 of a circle radius r applies to a circular field, the floor of a circular auditorium, or the cross-section of a circular tree trunk.
(ii) The same equation $v = u$ - at space that governs the motion of a body starting with velocity u and having a constant retardation a, also governs the state v of a bank account in which u is originally lodged, and a is withdrawn at regular intervals.
(iii) The same differential equation describes the motions of light waves and electromagnetic waves. Since both waves travel at the same speed, Maxwell deduced that both waves are in essence identical.

That $\sqrt{2}$ is not a rational number is due to the Pythagoreans. The Greeks refused such numbers into their algebraic system, calling them irrational numbers. In order to think of irrational numbers with exactness, they represented numbers geometrically. This became complicated, and not much progress was made in algebra in general. However, the Greeks made lasting contributions in high arithmetic or the theory of numbers, and in geometry. An important result in the first group is a result of Euclid, still considered an example of elegance in Mathematics today - that there is no greatest prime number. For if n is such a number, and p is the product of all the primes, then the number p + 1, which is greater than n, is a prime number since the number 1 is a remainder when p+1 is divided by any of the primes. Not only are both the idea and proof easy to understand, but in addition, they are just as important and meaningful today as in Euclid's time.

According to G.H. Hardy, "If a young person did not appreciate the proof of Euclid's theorem that there is no greatest prime number, then indeed, he was blind to the charms of Mathematics".

The Greeks favoured geometry, and built up a huge structure, most of which Euclid included in his "Elements". Whatever their other contributions in Mathematics, such contributions must rank behind the development of logical procedures, the examination of evidence, the sifting of axioms into ten, and the combination into thirteen books, constituting Euclid's Elements, of 465 theorems based on the ten axioms. The geometry in Euclid's elements - Euclidean geometry is still essentially studied in schools today, and has accordingly exerted enormous influence in succeeding civilizations, not only supplying inspiration for mathematical activity, but moulding generations, in the direction of logical thinking.

The civilization that preceded the Greeks regarded nature as chaotic and mysterious. To them, the happenings in nature were manipulated by the gods, and prayers and magic induce the gods to be kind. From about 600 B.C., we find among Greek intellectuals, a new attitude towards nature: rational, critical, and secular. They saw man's value in terms of his rationality. They thought of man as a rational being living in a rational world operating by precise laws that were waiting to be discovered. They insisted that nature was rationally, and indeed mathematically designed, and that by reasoning, chiefly with the help of Mathematics, the mathematical design of nature could be obtained.

The chief culture centre during the classical Greek period was Athens. When Alexander the Great conquered Greece, Egypt, and the near East, and found Alexandria as his capital, he made deliberate effort to fuse Greek and Near East Cultures. Consequently, the civilization centre at Alexandria, though predominantly Greek, was strongly influenced by Egyptian and Babylonian contributions. The mixture of the theoretical interests of the classical Greeks and the practical outlook of the Babylonians and Egyptians is evident in the Mathematical and Scientific works of the Alexandrian Greeks. The purely geometric investigations continued, and indeed, Euclid lived in Alexandria, though his work reflects the achievements of the classical Greeks. Appolonius and Archimedes pursued their Mathematical and Scientific studies during the Alexandrian period. For practical applications, which usually require quantitative results, the Alexandrians revived the crude arithmetic and algebra of Babylonia and Egypt, and used these empirically derived tools and procedures along with results derived from exact geometrical studies.

One major achievement of the Greeks in the search for a mathematical design of nature is the Ptolemaic theory in astronomy. In this theory, the earth is the centre of the universe, and is stationary. To account for the motion of a planet, the theory goes that the planet moves at a constant speed along a circle whose centre at the same time also moves in a circle at a constant speed round the earth. The radii of the circles and constant speeds as above, were different for different planets, and were chosen to agree with observed positions of the particular planets. Ptolemaic theory was accepted as the true design of the universe and held sway for about 1,500 years. However queries were raised as to the difficulty of fitting the motions of all the planets to a system of two circles. In some cases, a system of three or more circles was used to explain the motion. In order to make the theory fit the increased amount of data available, the number of circles needed also increased. At some stage, seventy seven circles were used to account for the motion of the sun, moon, and five planets! Nicolans Copernicus (1473-1543) studied these modified versions of the ptolemaic law, and about 1530, he proposed a new system of astronomy, called the heliocentric theory. This theory was considerably improved by J. Kepler (1571-1630). In the heliocentric theory, the sun is fixed. The earth, like any other planet moves in a circle at a constant speed round the sun, and at the same time rotates at a constant speed round its axis. The heliocentric theory permits us to look at the motion of a planet in terms of one circle only - round the sun, in contrast to the ptolemaic theory, however, with the improvement by Kepler, the planets do not move in a circle. Each planet moves in an ellipse, and the sun is at one common focus of each of these elliptical paths. Further, the planets do not move at constant speed. Indeed in Kepler's third law, he established the equation T^2 - KD^3 between the period T of revolution of any planet, and its mean distance D from the sun.

A sharp break with the past, the work of Copernicus and Kepler had far reaching effect on the formation of modem culture. Having got used to the idea of a fixed earth for about 1,500 years in accordance with the ptolemaic theory, difficulties in readjusting thinking in the new theory led to many scientific questions being raised. In addition, there was the weight of religious thought against the new theory. The Christian theology was built on the notion that the universe revolves round man, and that man was God's most important creation, everything in the universe including sun, moon, stars being designed to serve him. It was very convenient for this theology that the earth was regarded as the centre of the universe, further suggesting that heaven was in the heavens, and that hell was in the interior of the earth which occasionally erupted through volcanoes, showing that hell fire really existed. The new theory placed no significance on the earth viz-a-viz the other planets, and consequently there was no special focus on man. Also the arrival of the heliocentric theory was not long after the protestant revolution. At the time, both protestants and catholics had become alarmed by any movement that tended to undermine religious beliefs. Leaders of both faiths then joined to attack the heliocentric theory and its sponsors. When verbal attacks failed to discourage leaning and convictions in favour of the heliocentrc theory, the church applied the power and threat of the inquisition, nevertheless, the theory appealed to more and more people including astronomers and navigators.

Very helpful in the cause of the heliocentric theory at this time were the observations of Galileo, made with the newly invented microscope. He observed four moons rotating round the planet Jupiter. These observations showed that planets, other than the earth, had bodies revolving round them in space. He then claimed that it was equally likely for the earth to be moving and yet having a moon revolving round it. He noticed that there were more than seven moving bodies in the heavens, a number that had been earlier on claimed as sacrosanct on scriptural grounds. Galileo also saw mountains on the moon, and spots on the sun. These observations contradicted the belief that the heavenly bodies, unlike the earth, were perfect bodies. These observations of Galileo, while not proving the heliocentric theory as correct, helped to destroy earlier convictions standing in its way of acceptance. Galileo's support for the heliocentric theory angered the Roman inquisition. In 1616, he was called to Rome. His teaching of the heliocentric theory was roundly condemned by the inquisition, and he had to promise not lo publish anymore on the subject. In 1630, Pope Urban VIII gave him permission to publish, provided that Galileo would make his book mathematical, and not doctrinal. In view of this he published in 1632, only to be called again in 1633 by the inquisition. Under the threat of torture, he was forced to deny his advocacy of the heliocentric theory. He was forbidden to publish his material, and was virtually under house arrest. His manuscript was however smuggled out to Holland and published in 1638. By the middle of the 17th century, the scientific world had virtually accepted the heliocentric theory.

In Greek society, Mathematicians, Philosophers, Artists were members of the highest social class. This upper class either completely looked down on commercial pursuits and manual work or regarded them as unfortunate necessities. To them, such would reduce time available for intellectual and social activities amongst others. The Greek attitude to work might have had little influence on their culture were it not for the fact that they had a large slave class to whom they could pass the work. Slaves ran the business and the house holds, did unskilled and technical work, managed industries etc.

The slave basis of the classical Greek society encouraged development of the abstract side of Mathematics and science, with a consequent neglect of experimentation and practical application. However, about the 16th -17th century, time had changed from those of the classical Greeks. The introduction of the gunpower, muskets, and canons revolutionized methods of warfare, and gave a new social class of free men an important role. With the compass known in Europe, long range navigations became easier, resulting in more raw materials, belter trade routes, the discovery of America, influx of ideas into Europe etc.

Greek intellectuals believed that basic truths exist in the human mind. The purpose of their study of nature was lo determine the implications of these truths, to satisfy their curiosity and to organize their conclusions in patterns pleasing to the mind. In the 16th - 17th century, the general outlook of society had changed and so the purpose of the study of nature had also changed. The new goal of the study of nature as set out by Galileo (1564-1642) and Descartes (1596 -1650) was not only to understand nature, but also to make nature serve man. Mathematicians and scientists then sought facts not only from the mind as in the past, but also from engineers, technicians, etc. A new method for the pursuit of the truth of nature was gradually evolved: Experience and experiment were lo supply basic mathematical principles, and mathematics was to be applied lo these principles lo deduce new truths, just as new truths were derived from the axioms of geometry.

According to Galileo,
"Philosophy (nature) is written in the great book which ever lies before our eyes -I mean the universe - but we cannot understand it if we do not first learn the language and symbols in which it is written. The book is written in the mathematical language, and the symbols are triangles, circles, and other geometrical figures, without whose help it is impossible lo comprehend a single word of it, and without which one wonders in vain through the labyrinth".

According lo Descartes,
"Neither admits nor hopes for any principles in Physics other than those which are in geometry or in abstract Mathematics, because all phenomena of nature are explained and some demonstrations given".

Descartes insisted that the most fundamental and reliable properties of matter are shape, extension and motion in space and time. Since according to him, shape is just extension, he asserted. "Give me extension and motion and I shall construct the universe". These two seventeenth century philosophers, Galileo and Descartes revolutionized the very nature of scientific enquiry, altering the methodology of science. Because science was being asked to use quantitative axioms and mathematical deductions, the mathematical activities that were directly inspired by science became dominant.

One might for example, determine by some geometrical argument what type of curve a projectile from a canon follows, but geometry could not possibly answer such questions as how high or how far the projectile would go. The search for new methods in Mathematics that would be more efficient in giving numerical results led to intense focus on arithmetic and algebra. Descartes and Fermat established a system called coordinate geometry, whereby points of a curve are associated with certain ordered pairs of numbers called coordinates; an algebraic relation between these coordinates is called the equation of the curve. By specifying that a curve is any locus that has an algebraic equation, Descartes and Fermat provided an algebraic tool for studying the physical world, since through it, we now have a representation of geometrical figures by algebraic equations, and studies of these algebraic equations yield properties of the curve. In addition, algebra supplies quantitative knowledge. In the example just mentioned of a projectile from a canon, suppose we are given that the initial speed of firing is U in a direction making ∞[9] with the horizontal. By applying physical laws, one obtains the horizontal and vertical distances (x(t), y(t) travelled after time t as follows

$$x(t) = (U\ COS\ \infty)\ t,$$

$$y(t) = (uSIN\infty)t - \tfrac{1}{2}gt^2$$

At this time, the relative horizontal and vertical speeds are $U\ COS\infty$, and $U\ Sin\infty - gt$. At the highest point, the relative vertical speed is zero, yielding $t = \frac{u}{g}.\ Sin\infty$, and

$$y(t)\ \ u\frac{U^2}{g}\ Sin^2\infty\ -\ \frac{I}{2}\ g\ \frac{U^2\ Sin^2\infty}{g^2}\ = \frac{1}{2}\ \frac{U^2}{g}\ Sin^2\infty$$

However, at the furthest point, $y(t) = (u\ Sin\infty)\ t - \frac{1}{2}\ gt^2 = 0$

yielding $t = 2\ \frac{U\ Sin\infty}{g}$, and $x(t) = 2\ \frac{U^2}{g}\ Sin\infty\ \ Cos\infty$

Finally, from the expressions above, for x(t), y(t) at time t, we get the equation.

$$y = (\tan\infty)\ x\ -\ \left(\frac{g}{2U^2\ Cos^2\infty}\right) x^2$$

of the curve on which the projectile moves, While seeking fundamental quantitative physical principles for description of physical phenomena, Galileo introduced the all important concept of a function. After this had been done, the next two centuries witnessed considerable mathematical activity in constructing examples of functions, and studying their properties. Most of these examples were tied to physical situations, and both the functions and physical situations were studied together. Newton (1642 -1727) considered the area z under the curve as given by a function $z = a\,x^m$, where m is an integer or a fraction. He considered an infinitesimal increase ∂x in x, so that if y is the ordinate corresponding to the abcsissa x, then the new area is given by

$$z + y\,\partial x = a(x + \partial x)^{m}$$

He then applied the binomial theorem to the right hand side. On subtracting $z = ax^m$, dividing through by ∂x, and neglecting terms that still contain dx, he obtained

$$y = max^{m1}$$

In this process, Newton not only gave a general method for finding the instantaneous rate of change of one variable with respect to another, but also showed that the area can be obtained by reversing the process of finding a rate of change.

Eudoxus (408 - 355 B.C.) a member of the Platonian School of Classical Greece, had originated what eventually came to be known as integral calculus, through the Method of Exhaustion for computing areas bounded, not by polygons, but by more general curves. We inscribe in the domain, an approximating domain with a polygonal boundary, and thus an easily calculated area. By choosing another polygonal domain which includes the former, we obtain a better approximation to the given domain. Proceeding in this way, we gradually "exhaust" the whole area, and we obtain the area as the "limit" of the areas of a properly chosen sequence of inscribed polygonal domains with an increasing number of sides. This comes to treating the area as the Limit of sums. Eudoxus and Archimedes applied this method to special areas (circle, parabolas etc.). During the 17th century, many more cases were successfully treated. In each case, the actual calculation of the area was made to depend on an ingenious device specifically suited to the particular problem. One of the main achievements of the calculus was to replace these special and restricted procedures by a general and powerful one.

Combining the work of Newton as above, and the Eudoxus method of exhaustion, yielded the fundamental theorem of the calculus, which seeks to relate the process of integration as that of finding limits of sums, to a process of integration as one opposite to differentiation. Thus, the basic concepts of the calculus - the derivative and integral were introduced and their relationship established. Leibnitz (1646 - 1716) also independently obtained similar results about the same time as Newton, leading to controversy as to priority, and then polarisation between British and continental Mathematicians. Both Newton and Leibnitz must be credited with seeing the calculus as a new and general method, applicable to many kinds of functions. After their work, the Calculus was no longer an appendage or extension of geometry. By building on algebraic methods, they not only had a more effective tool than geometry, but it also permitted many geometric and physical problems to be treated by the technique. In obtaining the Fundamental Theorem of the calculus, they reduced the problems of instantaneous velocity, tangents, maxima, minima, and summation to differentiation and its inverse process - integration.

The immediate association of the calculus with geometry (tangents, lengths of curves), Mechanics (motion) etc., enabled the subject grow naturally in several directions in the 18th century leading to the subsequent opening up of such specialized areas as ordinary and partial differential equations, infinite series, calculus of variations, differential geometry etc., and the domain referred to as "Mathematical Analysis" was built up.

An assessment of the extent of contributions of Newton to Mathematics, may be summed up by a remark of his arch rival in the calculus controversy, Leibnitz, "Taking Mathematics from the beginning of the world to the time of Newton, what he has done is much the better half".

Inspite of all the growth in mathematics, the work of Newton and Leibnitz was heavily criticized at the time, from the angle of logical rigour. While their new invention, the calculus, led to correct results, their proofs and procedures were criticized as unsound and lacking in rigour. For example, neither Newton nor Leibnitz in the tradition of the Greeks rigorously defined his fundamental concepts of the calculus - the derivative and the integral. The concept of a function had to be put on a firm basis. Before the calculus, there were five common algebraic processes: addition, subtraction, multiplication, division, and extraction of roots. The work of Newton and Leibnitz in the calculus - obtaining areas, volumes, length of curves etc. involved a new algebraic process - the limit process for which algebraic tools had not been developed to put the subject on a firm logical basis. Rigorous axiomatic constructions gave way to induction from particular examples, intuitive insights, loose geometrical evidence, and physical arguments. Apparently, the mathematicians were abandoning the hallmark of their subject - deductive proof. The truth is that most of the mathematicians of the time were basically scientists concerned with major and pressing problems of science, and the mathematics they employed handled those scientific problems. However, the need to develop a strong foundation for the calculus was soon felt, Bolzano (1781-1848) and Cauchy (1789-1857) initiated a move in this direction. The concepts of a function, and limit of a function were defined. A real valued function f of a real variable x is a rule which associates a real number f(x) with each real number x. A real valued function f of a real variable x is said to tend to a limit 1 as x tends to x° if, corresponding to any positive real number t, there can be found a positive number ∂ such that whenever o < 1x - xol < ∂, we necessarily have that I f(x) - LI < t. If in this situation, f(x°) exists and is L, then the function F is said to be continuous at x°. The derivative of a function f at x (if it exists (here) is defined to be the limit of $\frac{f(x)\ f(x^0)}{x-x^\circ}$ as x (ends to x°.

The Riemann (1826-1866) integral may also be defined to be limit of sums of terms - which terms are essentially algebraic expressions for small areas in the Eudoxus Method of Exhaustion.

Earlier on, a continuous function was usually taken as one whose graph could be drawn without lifting pencil from paper. And a differentiable function was looked on as one for which tangents could be drawn to the graph. These earlier conceptions of continuous functions, differentiable functions, and integrable functions, depending as they were on areas, tangents to curves etc., rested on geometrical intuition, in contrast to the modern one depending on algebraic representation through the notion of a limit. Geometrical intuition suggested, for example, that a continuous function is differentiable and integration appeared limited to continuous functions. However, Weierstrass (1815 -1897) gave an example of a function, continuous everywhere, but nowhere differentiable;

$$f(x) \quad - \quad \sum_{n = o}^{\infty} b" \ \text{Cos}\ (a"\pi x).$$

where a is an odd integer, and b is a positive real number such that $ab > 1 + \frac{3\pi}{2}$

Riemann gave an example of a function which is discontinuous an infinite number of times in every interval but which has an integral that is continuous. Moreover this integral is not differentiable in an infinite number of points in any interval. This function is given by

$$g\,(x) = \frac{h\,(x)}{1} + \frac{h\,(x)}{4} + \frac{h(x)}{9} + \ldots\ldots\ldots\ldots.$$

where h(x) denotes the difference between x and the nearest integer except that h (x) is zero if x is halfway between two integers. The discovery of these weird functions violating laws which had been deemed perfect, were in some quarters looked on as signs of anarchy and chaos which mocked the order and harmony that previous generations had sought . According to Charles Hermite, “I turn away with fright and horror from this lamentable evil of functions which do not have derivatives”.

The reaction of Henri Poincare to these functions is similarly summed up by his observations as follows, “Logic sometimes makes monsters. For half a century, we have seen a mass of bizarre functions which appear to be forced to resemble as little as possible honest functions which serve some purpose. More of continuity, or less of continuity, more of derivatives, and so forth. Indeed, from the point of view of logic, these strange functions are the most general; on the other hand those which one meets without searching for them and which follow simple laws appear as a particular case which does not amount to more than a small corner. In former times when one invents them purposely to show up defects in the reasoning of our fathers and one will deduce from them only that”.

Inspite of all these feelings of surprise and reservation, the discovery of these pathological properties of certain functions: that continuous functions need not have derivatives, that discontinuous functions can be integrated etc., made Mathematicians realise that the study of functions extends beyond those used in the calculus and the usual branches of analysts where the requirement of differentiability usually restricts the clan of functions. The work of Bolzano, Cauchy, Weierstrass and others freed the calculus and its extension from all dependence upon geometrical notions, and intuitive understanding. Usually referred to as the “arithmetization of analysis”, this development was one of the profound mathematical events of the last century. Important bye-products of these efforts include the establishment of a logical basis for the real number system and the development of the theory of infinite sets, a subject arousing much controversy since Greek times.

In the preface to his Principia, Newton says, "Since the ancients (as we are told by Pappus) esteemed the science of mechanics of greatest importance in the investigation of natural things, and the moderns, rejecting substantial forms and occult qualities, have endeavoured to subject the phenomena of nature to the laws of mathematics, I believe this treatise cultivated mathematics as far as it relates to philosophy (science), and therefore I offer this work as the mathematical principles of philosophy, for the whole burden in philosophy seems to consist in this - from the phenomenon of motions to investigate the forces of nature, and then from these forces to demonstrate the other phenomena.......". This summarises Newton's views of the function of science.

The extent of his contributions in seeking a description of nature may be summed up by the following well-known saying, "Nature and nature's laws lay hid in the night God said, "Let Newton be", and all was light". In Newtonian mechanics for example, Newton showed that the law of gravitation, together with the two laws of motion give a constant acceleration for bodies falling near the surface of the earth, as well as a description of planetry motion in consonance with the heliocentric theory, and that, in addition, Kepler's laws of motion follow mathematically from them. In particular both terrestrial and planetary motion follow from one set of physical laws: the laws of gravitation, and the two laws of motion. By applying these, man can now create satellites which circle the earth. Newton got due recognition from his country. He was honoured with a Knighthood. And when he died, he was buried in Westminster Abbey with such pomp and Voltaire, who attended the funeral, said later, "I have seen a Professor of Mathematics, only because he was great in his vocation, buried, like a king who had done good to his subjects". The progress made in the 17th - 18th century led to more vigorous pursuits of science in the last century. One major scientific development then, which also stimulated mathematical activity was the study of electricity and magnetism. Physical principles were expressed mathematically, and mathematical techniques were applied to get new information, as Galileo and Newton did in their study of motion.

As mentioned earlier, the last century saw the "arithmetization of analysis". The last century also saw amongst others, the rise of abstract algebra. In particular, W. Hamilton, created a new algebra, by discarding the commutative postulate for multiplication. The introduction of the non-commutative quaternions was in itself significant, but perhaps its larger significance lay in the consequent discovery of the tremendous freedom that mathematics enjoys to build algebras that need not satisfy the restrictions imposed by the so called "fundamental laws". In this direction, amongst the mathematical developments of the last century, perhaps the most profound in intellectual significance was the creation of non-Euclidean geometry.

Euclidean geometry had ten axioms. Lets state five of them as in Euclid's Elements.

1. A straight line can be drawn from any point to any point.
2. A finite straight line can be produced continuously in a straight line.
3. A circle can be drawn with any centre and radius.
4. All right angles are equal.
5. If a straight line falling on two straight lines makes the interior angles on the same side together less than two right angles, then the two straight lines, if produced indefinitely meet on that side on which the angles are together less than two right angles. It was noted early that this

fifth axiom lacks the simplicity of the first four. Further its converse is proved in the "Elements". It was therefore felt that the fifth axiom should not be an axiom, but should be derivable from the other nine axioms.

The fifth axiom is equivalent to the following:
Through a given point not on a given line, there is at most one line parallel to the given line.
The fifth axiom is therefore referred to as the parallel axiom. Effort dating back to Greek times to reduce the number of axioms from ten to nine, by removing the parallel axiom failed, and in the last century, Gauss (1779 -1855), Bolyai (1802 -1860), and Labochesky (1793-1856) independently replaced the parallel axiom by the assumption that more than one parallel line to a given line can be drawn to pass through a given point. They went on to develop a geometry - naturally referred to as a non-Euclidean geometry, a geometry unrelated to the every day world of field, plans, boundaries etc. from which the original geometry started in ancient Egypt, but a geometry which nevertheless forms some picture of the world. In this geometry, the results of Euclid which are independent of the parallel axiom are still true. Moreover, in the new geometry for example the sum of the angles of a triangle is less than 180°, this sum varies with the area of the triangle in such a way that the smaller the area, the nearer is the sum of angles to 180°.

Riemann instead, replaced the parallel axiom by proposing that any two straight lines in a plane meet He also changed axiom-2 above of Euclid. Riemann then constructed a geometry, in which all straight lines have the same length, the sum of the angles of a triangle exceeds 180°, decreasing to 180° as the area of »triangle decreases to zero. The non-Euclidean geometry of Riemann applies directly to the surface of a sphere, provided that a "straight line" is interpreted to mean a ''great circle on the sphere". That there can be geometries other than Euclid's was in itself a remarkable discovery, and like the heliocentric theory, it had tremendous effect on the history of thought. The question that followed their discovery was whether these new geometries are of use in physical interpretations. In the theory of Relativity, Albert Einstein in this century employed a non-Euclidean geometry,'his predictions had wider areas of validity than in the older theory of Newtonian mechanics based on Euclidean geometry. Also, the non-Euclidean geometry of Riemann supplies answers to practical and scientific problems involving geometrical relationships on the surface of a sphere, and hence it is a geometry of the physical world, the earth being regarded as spherical. Mathematicians and scientists had sought to understand the physical world by adopting axioms which seem to fit it, and then deduce theorems from these axioms. It is now clear that unlike the insistence by Descartes in his philosophy of science, there is no reason to identify the mathematical construction with physical space, since several different mathematical theories may fit equally well. This mathematical theory of space is like other scientific theories, the one used is that which fits best at the time.

The original concepts of Mathematics, for example, straight lines (the natural numbers: 1,2,3,........ were immediate idealizations of, or abstractions from experience. The negative numbers, the irrational numbers are not, but they eventually gained acceptance. By the 17th -18th century, more of these concepts that have no immediate counterpart in the real world gained acceptance, forcing the recognition that mathematics is afterall a human, somewhat arbitrary, creation. With the creation of the non-commutative quarternions, and non-Euclidean geometry last century, the view that Mathematics can introduce and deal with rather arbitrary concepts was strengthened. Thus, the view point emerged that the subject matter of mathematics is afterall not the study of numbers or space or elaborations thereon, but simply the determination of consequences of systems of axioms.

There have, of course, been valuable contributions to the development of mathematics not mentioned so far. For example, the Indians (Hindus) conceived of the number zero, negative numbers, the decimal numeral systems etc. There are contributions from Arabs and Chinese. Indeed, whereas it is true that during the ancient tones and middle ages, the Greeks, and then die Arabs stood head and shoulder above others, in the modem period no national group remained the leader for any prolonged period. And the centre of mathematical activity shifted repeatedly from Germany to Italy to France to Holland to England etc. Modem mathematical activity seems now to be primarily inspired by European thought, and by men who studied in Europe and America and helped build up centres in their own countries. We seem to fall into this latter category ?As at now, do we have enough to show? If not, why not and in this case, what is the way ahead like? Perhaps a glimpse at the history of the development of mathematics could be useful, together with knowledge of paths taken by others who were similarly placed in the recent past, as well as the capacity and the will to meet wishes with effort.